

Chapter 5
Section 5.4

Def: If α is an angle whose terminal side passes through the unit circle at (x, y) we define the **tangent, cotangent, secant, and cosecant** functions as:

$$\tan(\alpha) = \frac{y}{x}, \cot(\alpha) = \frac{x}{y}, \sec(\alpha) = \frac{1}{x}, \text{ and } \csc(\alpha) = \frac{1}{y}$$

Alternate Def: Recall if α is an angle whose terminal side passes through the unit circle at (x, y) , then $\sin(\alpha) = y$ and $\cos(\alpha) = x$. This gives us alternate definitions for the above functions by:

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}, \cot(\alpha) = \frac{\cos(\alpha)}{\sin(\alpha)}, \sec(\alpha) = \frac{1}{\cos(\alpha)}, \text{ and } \csc(\alpha) = \frac{1}{\sin(\alpha)}$$

Ex: Find the value of all six trigonometric for $\alpha = \pi/4$ and $\beta = 150^\circ$

$$\sin(\pi/4) = \frac{\sqrt{2}}{2}$$

$$\cos(\pi/4) = \frac{\sqrt{2}}{2}$$

$$\tan(\pi/4) = 1$$

$$\csc(\pi/4) = \sqrt{2}$$

$$\sec(\pi/4) = \sqrt{2}$$

$$\cot(\pi/4) = 1$$

$$\sin(150^\circ) = 1/2$$

$$\cos(150^\circ) = -\sqrt{3}/2$$

$$\tan(150^\circ) = -\sqrt{3}/3$$

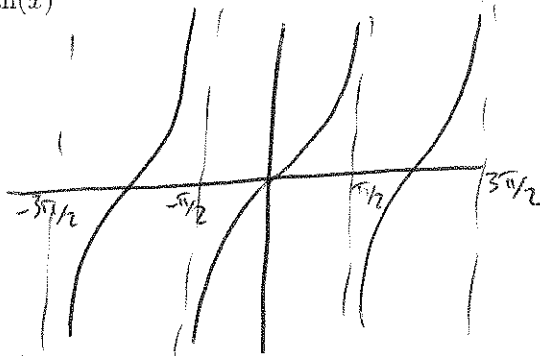
$$\csc(150^\circ) = 2$$

$$\sec(150^\circ) = -\frac{2\sqrt{3}}{3}$$

$$\cot(150^\circ) = -\sqrt{3}$$

Q: What are the domains of these new functions?

Graph of $y = \tan(x)$



Period = π

Amplitude undefined

Dom = $\{x \in \mathbb{R} : x \neq \pi/2 + k\pi (k \in \mathbb{Z})\}$

Ran = $(-\infty, \infty)$

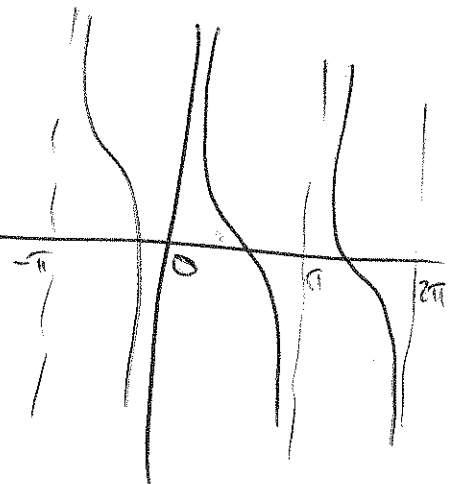
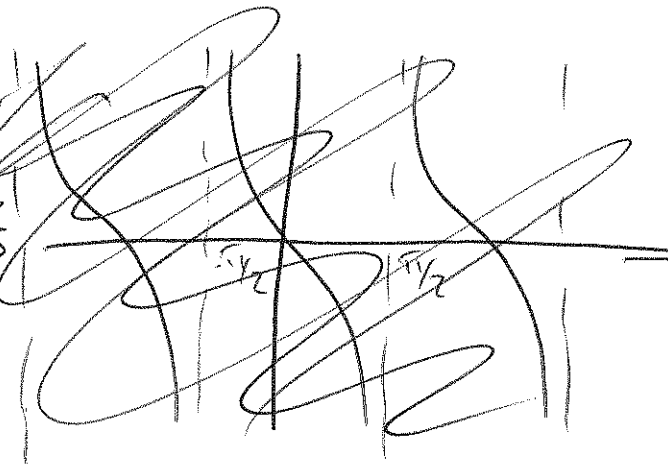
Graph of $y = \cot(x)$

Period = π

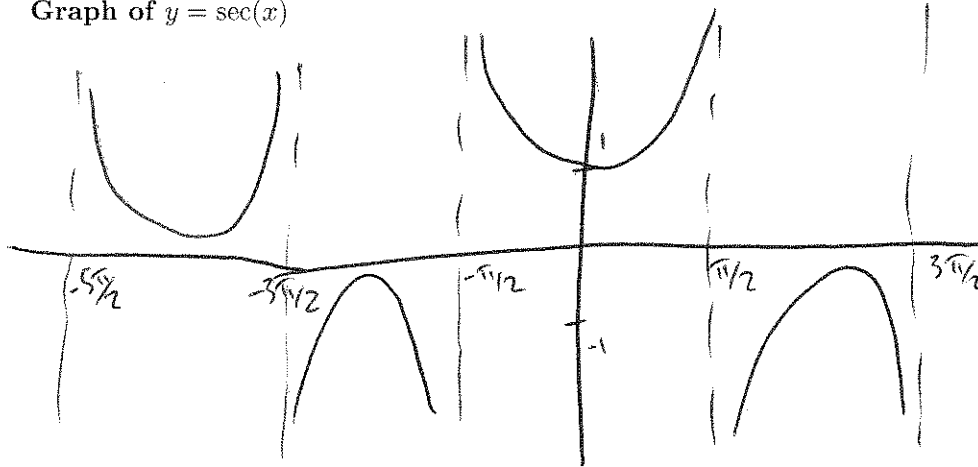
Amp undefined

Dom = $\{x \in \mathbb{R} : x \neq k\pi (k \in \mathbb{Z})\}$

Ran = $(-\infty, \infty)$

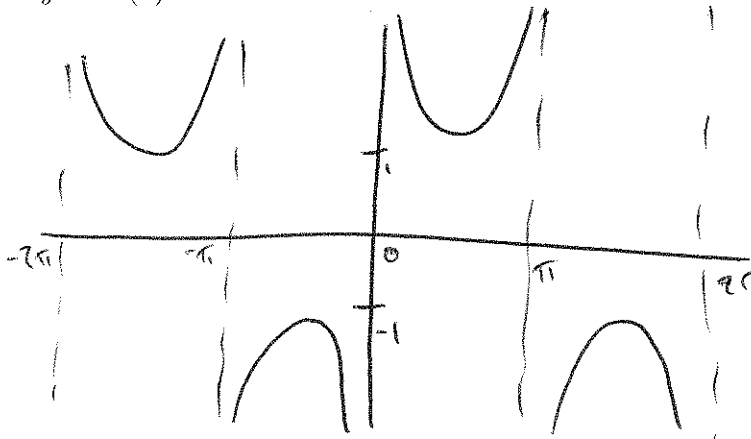


Graph of $y = \sec(x)$



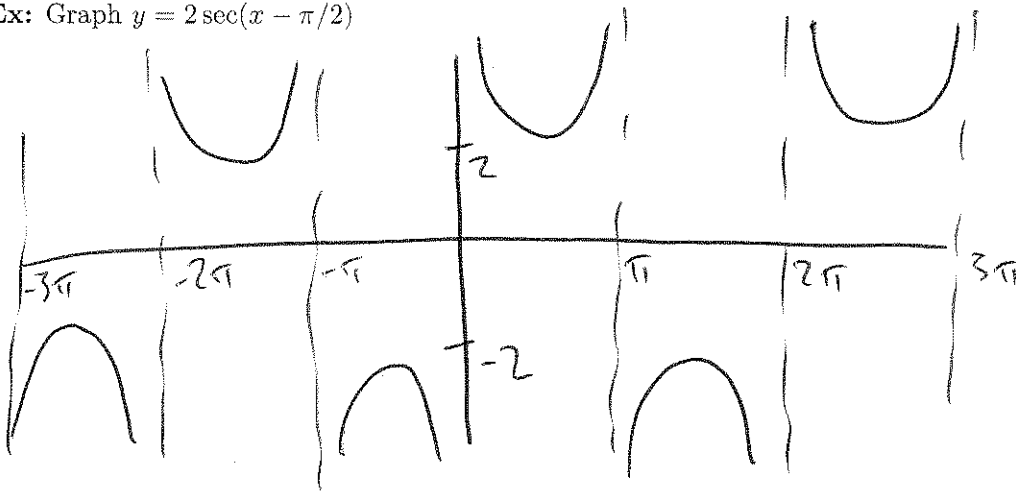
Period = 2π
 Amp undefined
 Dom = $\{x \in \mathbb{R} : x \neq \frac{\pi}{2} + \pi k\}$
 Ran = $(-\infty, -1] \cup [1, \infty)$

Graph of $y = \csc(x)$



Period = 2π
 Amp undefined
 Dom = $\{x \in \mathbb{R} : x \neq k\pi (k \in \mathbb{Z})\}$
 Ran = $(-\infty, -1] \cup [1, \infty)$

Ex: Graph $y = 2 \sec(x - \pi/2)$



Practice: 8, 13, 18, 22, 55, 59, 73, 75